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ELECTRIC FIELD RESPONSE OF TWO-DIMENSIONAL INHOMOGENEITIES TO UNIPOLAR AND BIPOLAR ELECTRODE CONFIGURATIONS

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A quantitative comparative analysis of some bipolar and unipolar resistivity arrays is made in terms of perturbation effects due to buried lateral inhomogeneities. The Schlumberger, the collinear dipole-dipole, the pole-dipole, and the unipole electrode configurations of approximately the same dimensions are employed. A "focusing effect" in the vertical component of the current density distribution is demonstrated in the unipole configuration. Characteristic diagrams illustrating the configuration responses to lateral inhomogeneities located at different depths and under various overburden layer conductivities indicate that, for comparable geometric spreads, the unipole and the pole-dipole configurations are significantly more effective.

INTRODUCTION

During the last several decades, analytic and physical scale models have been developed simulating horizontal layer stratification, contacts, dyke shaped inhomogeneities, and other simple models (Apparao et al, 1969; Van Nostrand and Cook, 1966). A few studies dealing with the depth of exploration of different electrode configurations over layered earth models have been made (e.g., Muskat and Evinger, 1941; Roy and Apparao, 1971), but there has been very little analysis of the response of these electrode configurations to laterally inhomogeneous models. In recent years, the development of practically realizable numerical methods, such as, the finite-difference, finite-element, transmission-line, or integral equation methods, has led to the analysis of the response of any electrode configuration to inhomogeneities of arbitrary shape (Coggon, 1971).

In this paper a comparative analysis has been made of the response of the Schlumberger, collinear dipole-dipole, pole-dipole, and a relatively novel configuration the unipole, to the presence of some simple two-dimensional inhomogeneities. Two-dimensional structures are defined here as geologic bodies of arbitrary cross-section in the vertical plane and of infinite horizontal extent in the strike direction. The arrays are shown in Figure 1 with the standard nomenclature for current, potential, and spacings. To reduce the number of figures, we have omitted the Wenner configuration, basically because it has similar response characteristics to the Schlumberger.

In the three conventional arrays, e.g., Schlumberger, dipole-dipole, and pole-dipole, a potential difference is measured between the receiving electrodes in the absence of an inhomogeneity. The presence of buried lateral discontinuities results in relatively small perturbations of the primary field. In practice, an electrode configuration will be more effective the higher the ratio of the anomalous electric field ΔE, due to inhomogeneities, to the primary field E_p. The criterion for a comparative analysis should be

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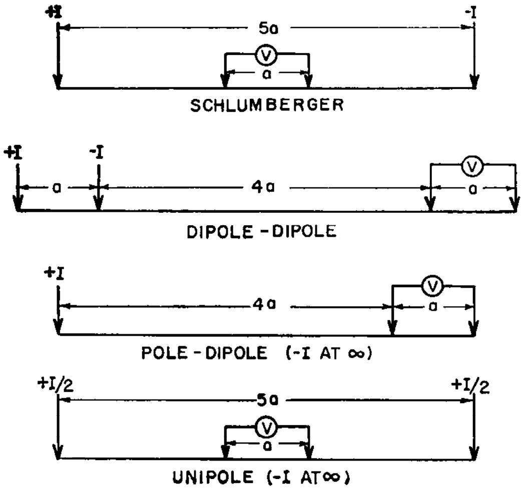

FIG. 1. Geometric configurations of the electrode arrays considered.

this ratio $\Delta E/E_p$ provided some way can be found to normalize the dimensions of the different arrays. In the unipole arrangement [Tarkhov (1957), Sakovtsev (1958), Gupta and Bhattacharya (1963), and Dey (1967)] shown in Figure 1, the current electrodes of a Schlumberger configuration carry equal current of the same polarity; the opposite polarity electrode is located at infinity. The primary field at the centrally located potential electrodes is thus zero for a homogeneous or horizontally layered lower half-space. This method is a "pure anomaly method" (Tarkov, 1957), since any signal measured at the potential electrodes reflects only a lateral resistivity contrast (the effect of errors in electrode spacings will be discussed later). Since there is no potential difference at the receiver electrodes for the layered or homogeneous case, there is no way of defining the apparent resistivity as in the conventional arrays. Thus, apparent resistivity cannot be used as the comparator in this study.

As mentioned above, a more viable scheme for comparing the arrays is based on the potential difference measured in practice across the receiving electrodes. It is first necessary to make some rather arbitrary decisions as to the dimensions of the arrays that are to be compared. We have chosen the arrays to be of approximately the same overall length, and we have chosen a, the length of the potential measuring dipole, to be identical for all the configurations. The current electrodes in the Schlumberger and unipole arrays are separated by a distance $5a$. The separation of the current dipole and potential dipole in the dipole-dipole array is held at $4a$, corresponding to a conventional dipole separation of $n = 4$. In the pole-dipole array, the dipole is located a distance $4a$ from the current pole. The polarity of the current electrodes in the unipole and pole-dipole configurations is assumed to be positive, the negative current electrodes being at infinity. The dimensions of the lateral inhomogeneities, the layer thicknesses, and the positions of the electrodes on a traverse are expressed in units of the potential electrode separation a. Whether the dipole-dipole and pole-dipole arrays should have had spacings of $4a$ or $5a$ to be better compared dimensionally to the other arrays is debatable. The comparative merits obtained below are the same whichever spacing is used.

CURRENT DENSITY ON THE CENTRAL VERTICAL PROFILE

The current densities developed by a bipole and a pole are well known (Van Nostrand and Cook, 1966; Gupta and Bhattacharya, 1963). For the unipole array, the current density distribution on the central vertical profile bisecting the current electrodes is shown in Figure 2. Because of the symmetry of the array, only J_z exists on the central profile. J_z is zero on the surface and zero at infinity, with a maximum at some finite depth. This is the "focusing effect" noted by Gupta and Bhattacharya (1963); it occurs at $Z/L = 0.75$ in a homogeneous half-space, where $2L$ is the separation of the positive current sources. On a stratified earth, the depth of focus increases as the lower layer becomes more conductive and decreases as the lower layer becomes more resistive. The saturation effect is prominent for $\rho_1/\rho_2 > 1$, in that for the unipole configuration, the amplitudes of J_z, while higher than those obtained over a homogenous earth, are substantially the same at $\rho_1/\rho_2 = 10$ and 30. For the cases with more resistive substrata, however, the amplitudes of J_z indicate remarkably different levels up to resistivity contrasts of 30.

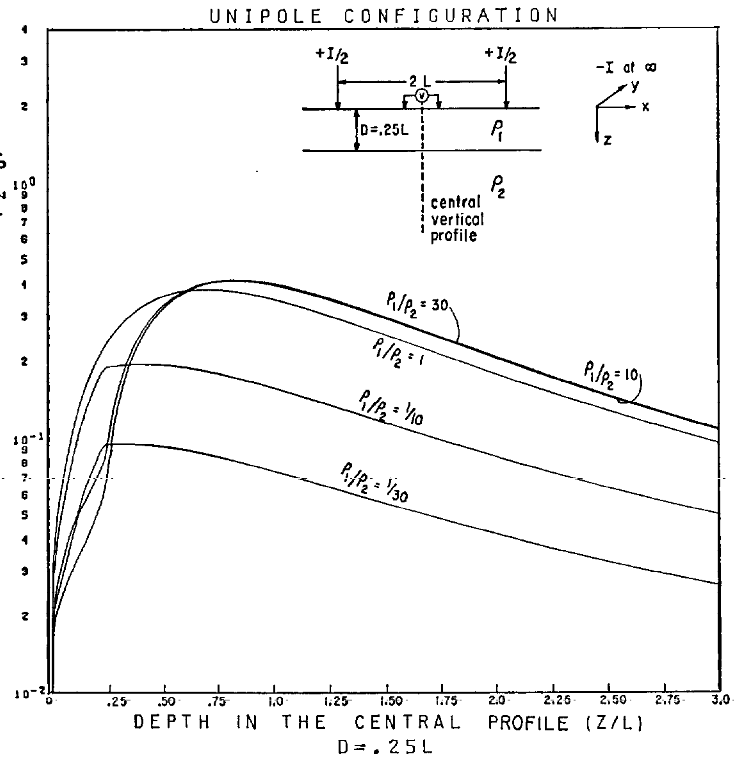

FIG. 2. Current density distribution in the central vertical profile for a unipole configuration of electrodes.

RESISTIVITY RESPONSE TO TWO-DIMENSIONAL INHOMOGENEITIES

Apparent resistivities measured over two-dimensional inhomogeneities of arbitrary cross-section and conductivity contrast with respect to the surrounding half-space can be numerically evaluated. In recent years, several numerical techniques have been developed by use of the transmission-line approach (Madden, 1967, 1971), the finite-difference method (Jepsen, 1969), and the finite-element technique (Coggon, 1971). In the present paper, the electric field responses of various electrode arrays to two-dimensional bodies are evaluated by numerical solutions obtained by the transmission-line method, and some of the results have been corroborated using the finite-element technique.

For purposes of comparison of the various electrode arrangements considered, a standard vertical, conductive, dike-shaped, two-dimensional inhomogeneity model has been adopted. The dike is assumed to have a thickness of $a/2$ and a depth extent large enough to be considered infinite. The dike is assumed to have a conductivity 100 times as great as that of the surrounding half-space.

In the conventional portrayal of the apparent resistivity response, the potential difference observed across the potential dipole is multiplied by the appropriate geometric factor. The geometric factors in the four arrays considered herein have a large range of values, and hence the apparent resistivity response amplifies the actual measured voltage by varying degrees for each configuration. A more useful scheme for comparison of the sensitivity of the various configurations would be based on the actual potential developed across the potential electrodes.

This voltage is represented as an "electrical intensity" in this paper, where we assume the potential dipole length a to be unity. The electric field intensity at the potential dipole is plotted after normalization by a constant

$$E_0 = \frac{I\rho_1}{6.25\pi a^2},$$

where E_0 is the intensity at the center of the Schlumberger configuration of electrodes located over a homogeneous earth of resistivity ρ_1. Since the dimensions of the arrays and inhomogeneities are all given in terms of a (the potential electrode separation), the normalization of the field intensities to E_0 (thus effected) makes the response to the inhomogeneity independent of the resistivity of the surrounding host rock (ρ_1) and the dimensions of the potential and, consequently, current dipoles. Any given set of results presented herein can thus be used for the given conductivity contrasts for proportionately larger and smaller targets by appropriate choice of the length of the potential dipole in the configuration used. The apparent resistivity responses in any of the configurations, except unipole, may be simply calculated by multiplying the plotted values of E_x by the appropriate geometric factors.

Figures 3, 4, 5, and 6 illustrate the normalized electric field intensity profiles computed for horizontal profiling over the standard dike model located at various depths for the Schlumberger, dipole-dipole, pole-dipole, and unipole electrode

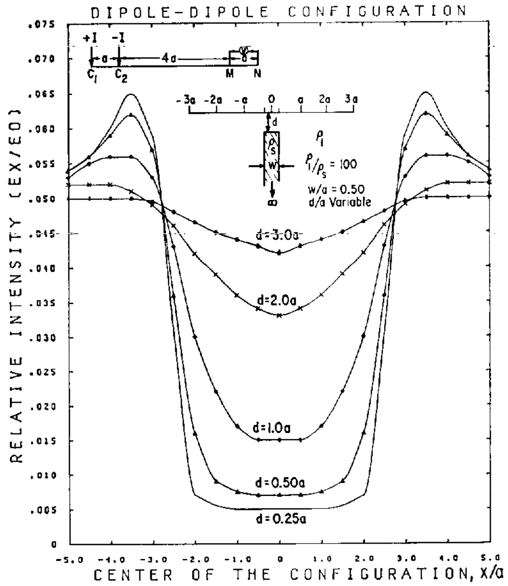

FIG. 4. Relative intensity profiles for a dipole-dipole configuration over a two-dimensional dike, for various dike depths.

arrangements, respectively. The profiles obtained with the Schlumberger arrangement (Figure 3) indicate remarkable changes in the spatial pattern as a function of the depth of burial of the conductive inhomogeneity. For very shallow

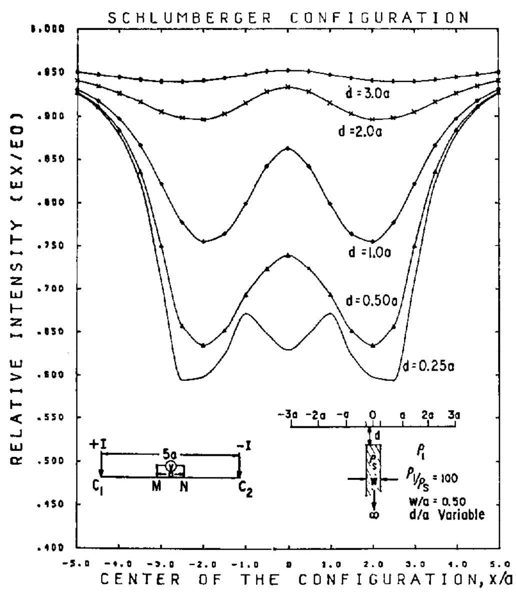

FIG. 3. Relative intensity profiles for the Schlumberger configuration over a two-dimensional dike, for various dike depths.

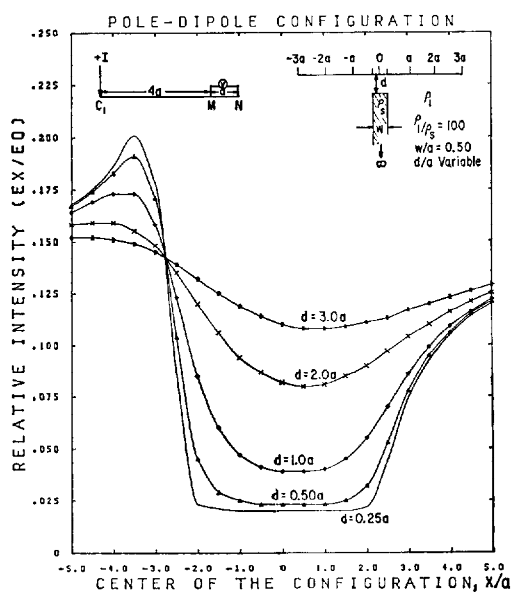

FIG. 5. Relative intensity profiles for a pole-dipole configuration over a two-dimensional dike, for various dike depths.

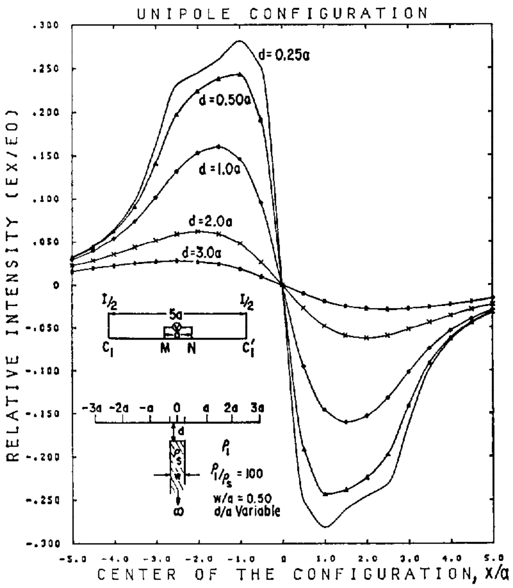

FIG. 6. Relative intensity profiles for a unipole configuration over a two-dimensional dike, for various dike depths.

dikes, the distinctive changes in curve shape are related to specific positions of the current and potential electrodes with respect to the position of the dike (Van Nostrand and Cook, 1966). For greater depths of burial, the major anomalies occur on either side of the dike with a small *relative high* directly over it. It should be noted that except for very shallow depths of burial, the lateral resolution achieved in locating the position of the dike from the profiles is poor.

The profiles obtained for the dipole-dipole arrangement of electrodes, as illustrated in Figure 4, indicate the position of the body by a marked *low* flanked by a *high* on either side. The generalized spatial pattern of the profiles remains more or less the same, albeit with progressively lower anomaly amplitude, for greater depths of burial. The responses obtained with the pole-dipole array also indicate the presence of the conductive inhomogeneity by *highs* on one side of the dike and by pronounced *minima*, as shown in Figure 5. It should be noted that although a minimum clearly indicates the presence of the conductive targets, the encompassing zone of the minimum is rather broad, thus lessening the lateral resolution achieved in locating the position of the target.

The response profiles for the inhomogeneity obtained with the unipole configuration, as illustrated in Figure 6, are rather remarkable. Although the response is identically zero over a uniform or layered earth, the perturbations due to the lateral resistivity discontinuity result in a simple spatial pattern that changes sign as the electrode configuration passes over the dike model. The line over the center of the target is located at the zero crossing of the profile. This pattern persists for dikes located at large depths, and the dike location is easily detected by the change in sign of the potential difference observed, even for low signal levels (Gupta and Bhattacharya, 1963).

For the unipole array, it is important to know the magnitude of the error in results due to errors in placement of the potential electrodes. Any deviation from the symmetrical placement of the electrodes would affect the response of the unipole array. To test the magnitude of this error, a "unipole" array was modeled with a potential receiver of standard length a but with its center displaced to the left so that 75 percent of the potential dipole length is to the left of the line bisecting the current electrodes. With this "unipole" array, a profile was calculated for the dike in Figure 6 with no overburden and $\rho_1/\rho_s = 100$. The resulting curve shape is the same as the curves in Figure 6 but with an amplitude error of 2 percent and a shift of the zero crossing a distance of $0.1a$ to the left. Therefore, the loss of symmetry due to a large error in placement of the potential receiver causes only a small error in the unipole profile.

CHARACTERISTIC CURVES FOR THE STANDARDIZED TWO-DIMENSIONAL INHOMOGENEITY

A generalized representation of the response to the different electrode configurations considered, for various depths of burial of the dike-shaped conductive inhomogeneity, is shown in Figure 7. The response profiles are analyzed in terms of the "anomaly index" variations with changes in the model parameters. The anomaly index, in percent, is defined as

$$A.I. = \left(\frac{E_x^{\max} - E_x^{\min}}{E_0}\right) \times 100.$$

The remarkably different magnitudes of the perturbation effect of the lateral inhomogeneity

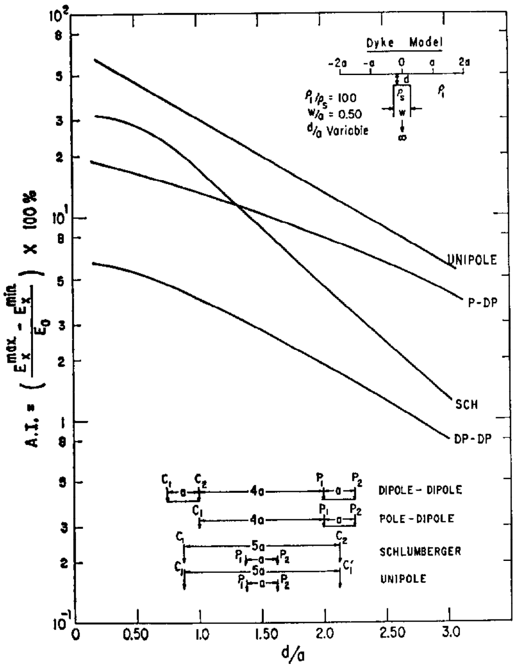

FIG. 7. Characteristic curves for the various configuration profiles over a two-dimensional inhomogeneity. Anomaly index versus depth.

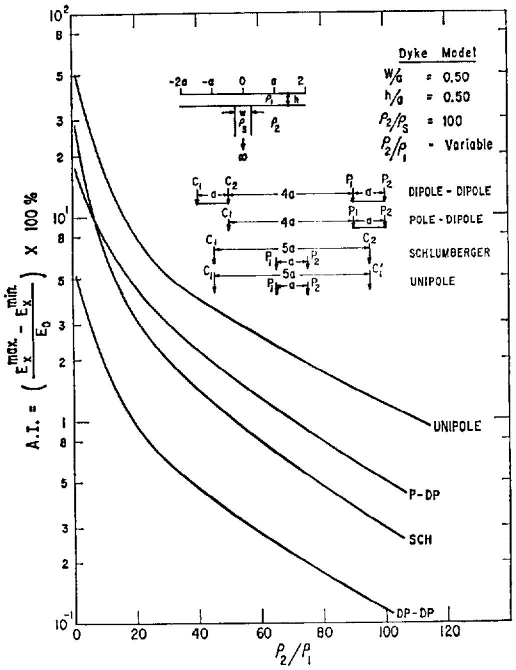

FIG. 8. Characteristic curves for the various configuration profiles with an overburden layer over a two-dimensional inhomogeneity. Anomaly index versus overburden conductivity.

at various depths of burial, for the different configurations, are clearly evident. For identical models, the anomaly index for the unipole configuration is the largest at all depths of burial. For shallow depths, the Schlumberger configuration shows a higher anomaly index than the pole-dipole; for larger depths, the pole-dipole index is substantially higher. The dipole-dipole index is the lowest at all depths. The detectability of the anomalies appears to be significantly higher for the unipole and the pole-dipole arrangements. The resolution of the depth of burial, however, seems greater for the Schlumberger configuration. A lower limit in the measurements of 1 percent *A.I.* prevents the effective detection of the dike at depths greater than $3a$ for the dipole-dipole and Schlumberger configurations, while at these large depths of burial, the unipole and the pole-dipole responses are approximately ten times as great in amplitude. If the measurable response from the two-dimensional conductive dike considered here is an indicator of the effective depth of search of the electrode configurations, it is evident that for comparable geometric spreads, the unipole configuration is the most efficient.

The effect of the overburden layer conductivity on the inhomogeneity response for the various configurations is shown in Figure 8. The standard dike model is assumed to be located directly under a conductive overburden layer of thickness $a/2$. On the logarithmic plot, the rate of decrease of the anomaly index with increased relative conductivity of the top layer is somewhat similar for the different configurations, except for very small contrasts in ρ_2/ρ_1. If a limiting sensitivity of 1 percent in the anomaly index measurement is assumed, the conductive dike is not detectable for a resistivity contrast (ρ_2/ρ_1) greater than 20 for the dipole-dipole and 50 for the Schlumberger array. The unipole and the pole-dipole configurations, however, have a remarkably high degree of penetration through the conductive overburden layer in that the target is detected (at the 1 percent level) with ρ_2/ρ_1 as large as 100. This significantly advantageous feature of the unipole and the pole-dipole configurations is explained by the unusually high percentage of current that enters the more resistive substratum and results

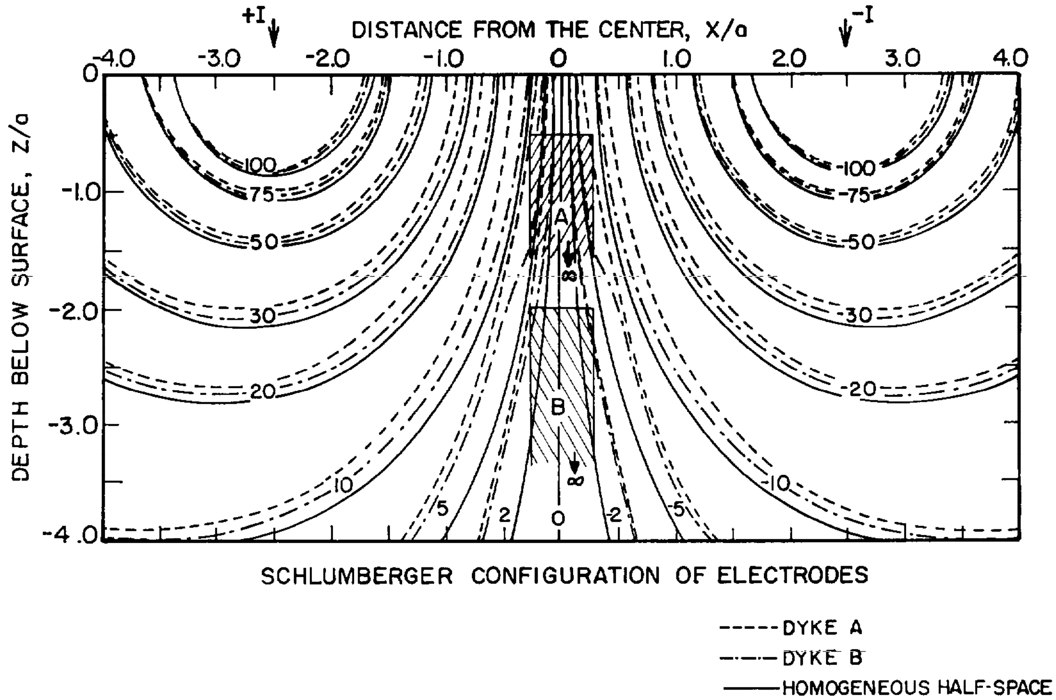

FIG. 9. Subsurface potential distributions for Schlumberger configurations on half-spaces with or without lateral inhomogeneities.

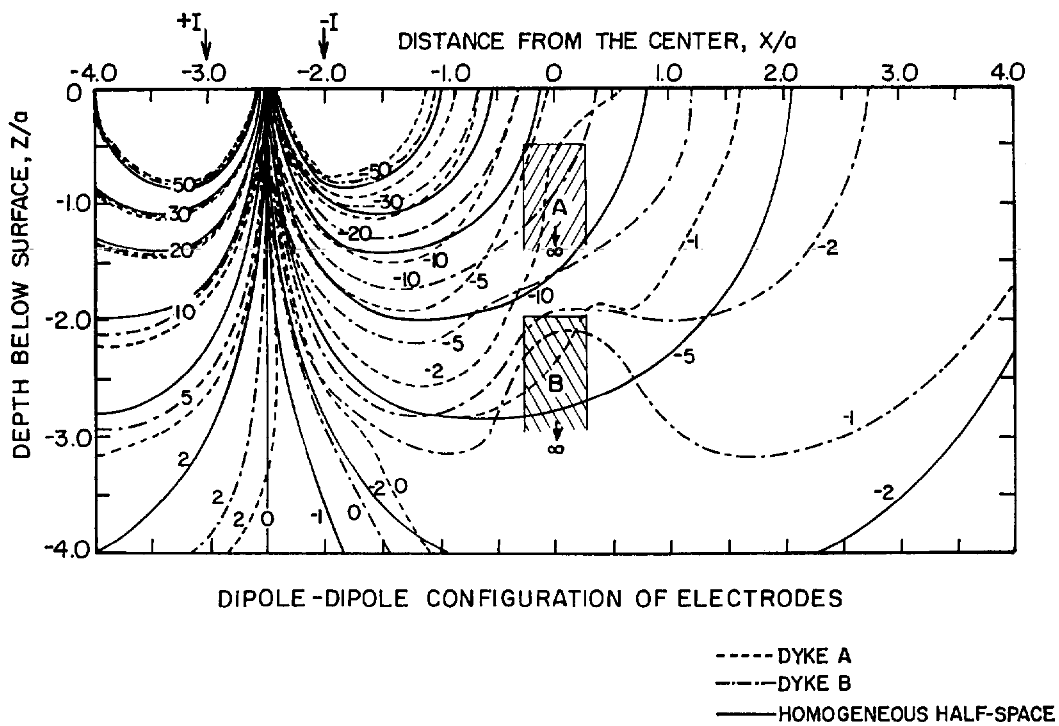

FIG. 10. Subsurface potential distributions for dipole-dipole configurations on half-spaces with or without lateral inhomogeneities.

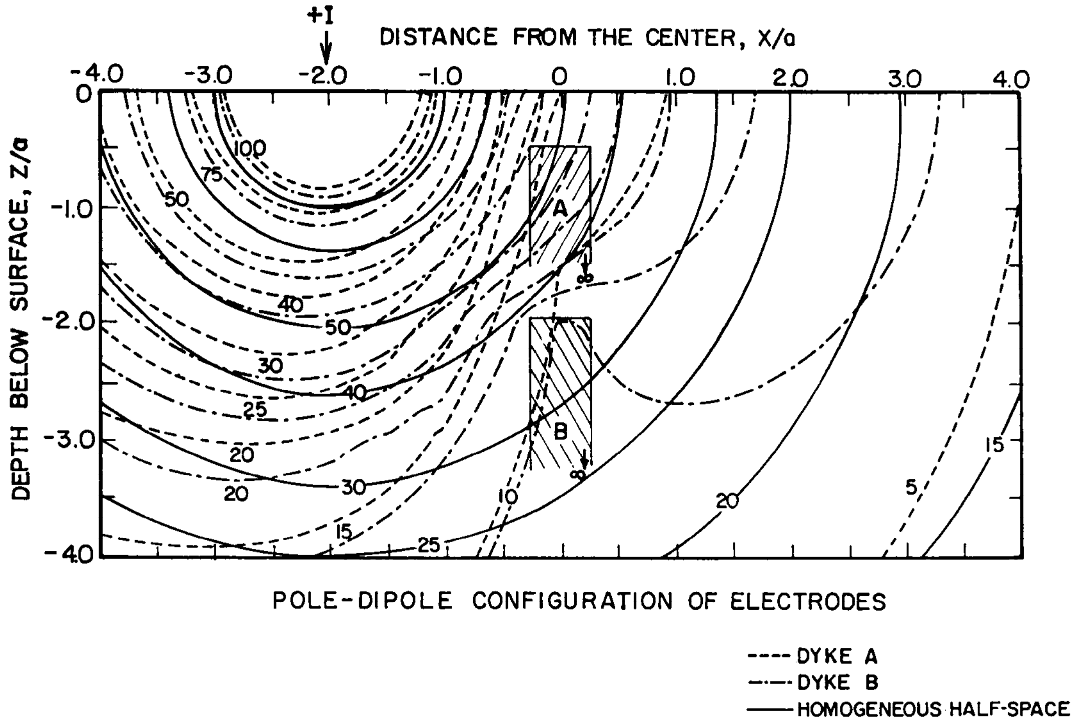

FIG. 11. Subsurface potential distributions for pole-dipole configurations on half-spaces with or without lateral inhomogeneities.

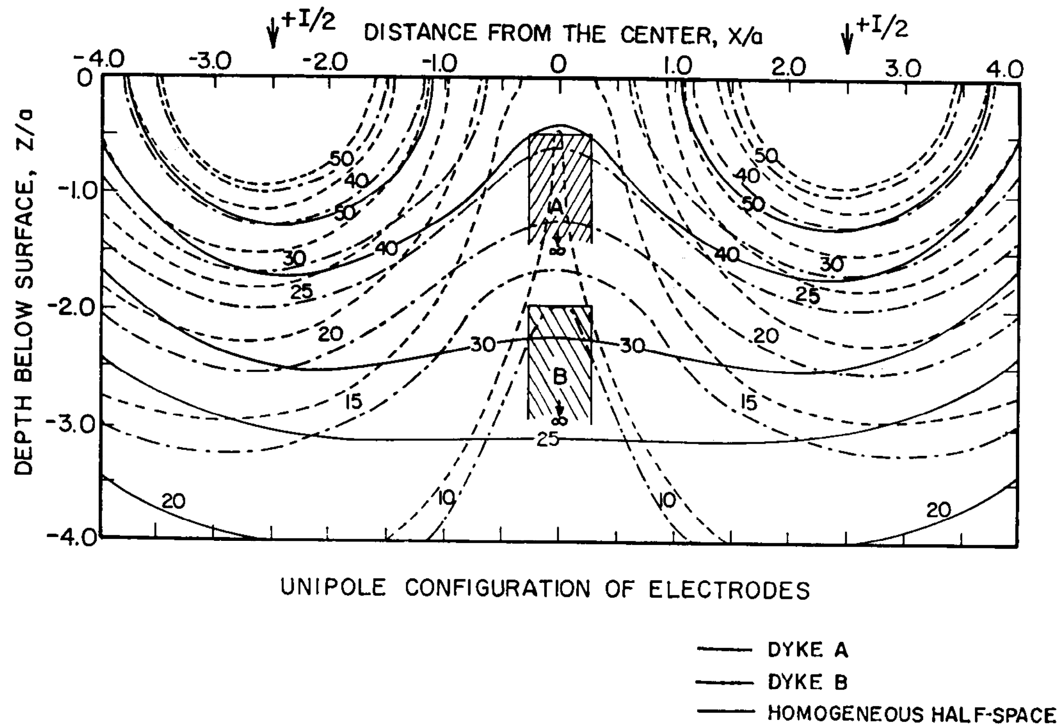

FIG. 12. Subsurface potential distributions for unipole configurations on half-spaces with or without lateral inhomogeneities.

in substantially higher perturbations due to the inhomogeneity.

POTENTIAL DISTRIBUTIONS IN THE LOWER HALF-SPACE

A portrayal of the subsurface potential distribution in the presence of the inhomogeneity is of interest. These subsurface potential distributions are illustrated for the various configurations considered in Figures 9, 10, 11, and 12. The potentials are contoured in arbitrary units. The contour levels are indicated, and the three different lines indicate the results obtained with the dike at two different depths and with the homogeneous earth. The results corresponding to two different depths of the standard model ($d = a/2$ and $2a$) are superposed over the response of the homogeneous half-space for purposes of comparison.

The perturbation effects of the conductive dikes appear to be significant for the Schlumberger array only at depths greater than $1.0a$ except in the zone directly above the shallow dike. The secondary potentials for the dipole-dipole and the pole-dipole arrays are substantially higher in the subsurface near the dikes and are also reflected on the surface above the inhomogeneity. For the unipole configuration, however, the dike produces rather significant perturbations of the homogeneous earth

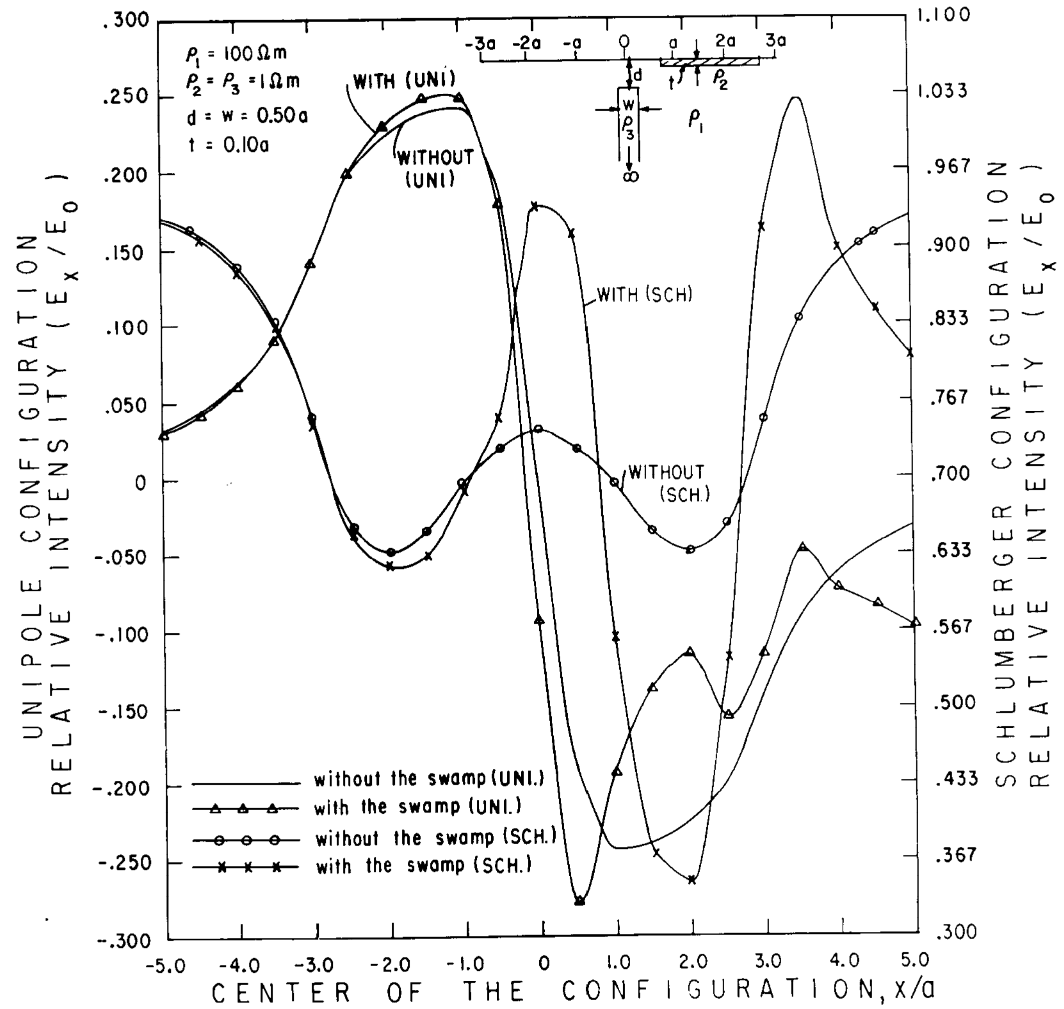

FIG. 13. Profiles for Schlumberger and unipole electrode configurations in the presence of a shallow, conducting noise source.

results at all levels in the subsurface. A study of the subsurface potentials for the different arrays indicates the unipole array, with the current electrodes on the surface, is the most suitable for a more detailed location of the conductive inhomogeneity by down-hole potential measurements.

EFFECT OF FLAT, SHALLOW CONDUCTIVE ZONES ON THE ANOMALY PATTERNS

In practice, the anomaly signature is often obscured by rather large perturbations from local, near-surface, conductive noise sources. A study of the effect of such a shallow noise source, e.g., a swamp, on the response of the standard dike model was made for all configurations considered. Figures 13 and 14 illustrate the combined effect of the swamp and the target inhomogeneity as well as the response of the inhomogeneity alone. It is evident from the results that the Schlumberger array is the most susceptible to such near-surface, conductive noise sources. The response amplitude pattern and the lateral resolution achieved in locating the deep conductive target are substantially altered. The unipole response, however, still indicates the position of the deep target without appreciable error, and the position of the swamp is also indicated as a minor perturbation with good lateral resolution. The dipole-dipole and

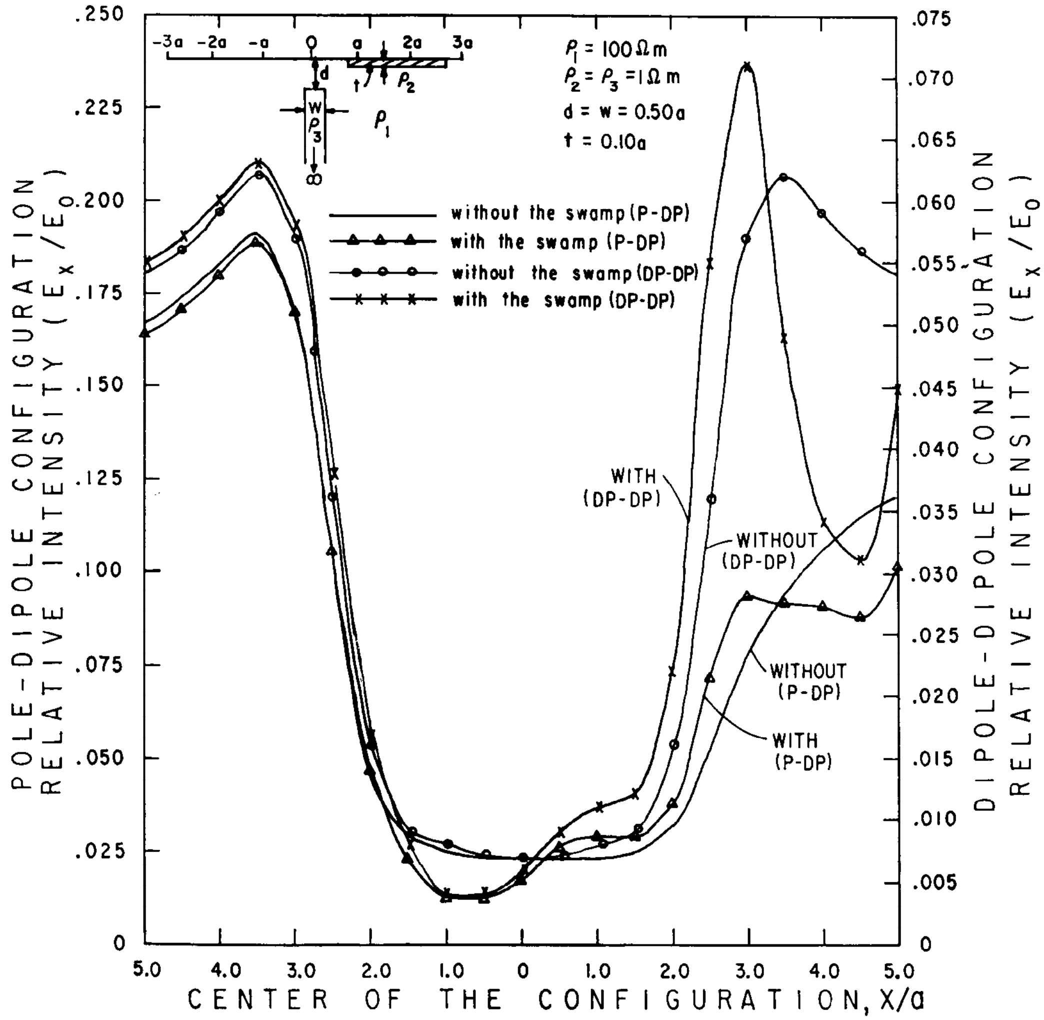

FIG. 14. Profiles for dipole-dipole and pole-dipole electrode configurations in the presence of a shallow, conducting noise source.

the pole-dipole responses are less affected than the Schlumberger, in that the broad minima still indicate the deep target location rather well. The effect of the swamp is larger for the dipole-dipole than for the pole-dipole. The secondary minimum in the dipole-dipole response is shifted in position with respect to the conductive swamp.

DISCUSSION AND CONCLUSIONS

A quantitative comparative study of bipolar and unipolar electrode configuration responses has been made in terms of the perturbation effects due to buried lateral inhomogeneities.

Although the unipole configuration studied here is insensitive to laterally uniform vertical discontinuities, it is shown to have remarkably good detection capability and resolution in delineating lateral inhomogeneities. The logistics of locating a current electrode at "infinity" may pose certain problems in its practical implementation in mineral exploration. In the southwestern U.S., however, the pole-dipole configuration is routinely used with large dipole separations. The unipole array can be concurrently implemented with a single additional current electrode, and, in the process, two pole-dipole profiles (or pseudo-sections) can be obtained, one with the current electrode leading and the other with the current electrode lagging the potential dipole. Such dual profiles of pole-dipole arrays are of immense practical importance in providing considerably higher resolution in the delineation of width, inclination, and depth of burial of dike-shaped targets. The vertical and horizontal subsurface resistivity distributions obtained with the pole-dipole configuraton can thus be substantially augmented by the high degree of lateral resolution of resistivity discontinuities provided by a simultaneous use of the unipole arrangement of electrodes.

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